A Family of Well-Clear Boundary Models for the Integration of Unmanned Aircraft Systems in the National Airspace System

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NASA Langley Research Center
in Support of
the UAS in the NAS Project

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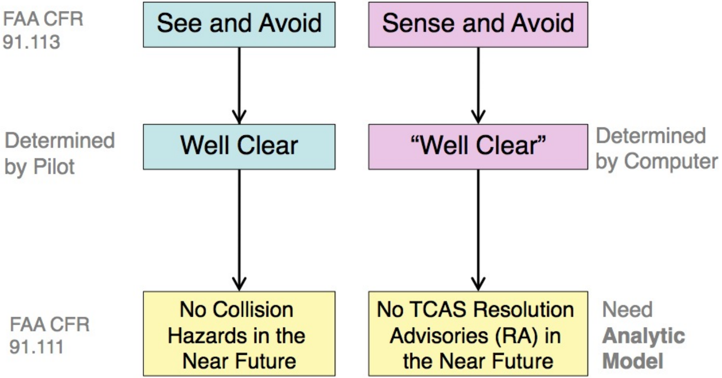
See and Avoid vs. Sense and Avoid

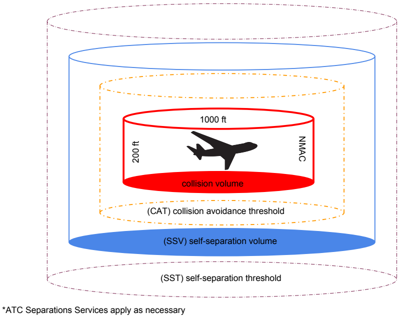


A Motivation for a Formal Definition of Well Clear

- The FAA SAA Workshop for UAS defines **sense and avoid** as: "the capability of a UAS to remain **well clear** from and avoid collisions with other airborne traffic."
- How will a UAS determine if it is **well clear** from other airborne traffic?
- In the absence of an on-board human pilot with the **experience** and **judgement** to determine well clear, a formal definition is needed to provide guidance to a ground pilot or possibly an automated algorithm.
- This definition should be more **conservative** than TCAS, a system intended to be the last resort in collision avoidance, so as to be compatible.
- NASA has examined and developed several formal definitions which considered to be a **family** of **well-clear boundary models**.

The Approach

A key characteristic of NASA's concept is that the self-separation threshold is a conservative extension of the collision avoidance threshold defined by TCAS.[1]



Volumes and thresholds are shown as cylinders for illustrative purposes only. In general, these shapes are irregular, with the exception of the collision volume.

[1] Consiglio, Chamberlain, Muñoz, and Hoffler, ICAS, 2012

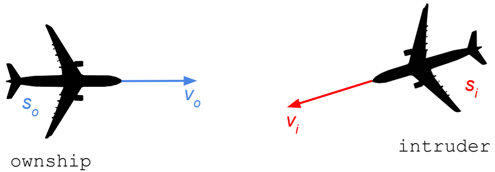
Interoperability with TCAS RA Logic

- TCAS is a family of airborne devices that are designed to reduce the risk of mid-air collisions between aircraft equipped with operating transponders. TCAS II, the current generation of TCAS devices, is mandated in the US for aircraft with greater than 30 seats or a maximum takeoff weight greater than 33,000 lbs,
- To ensure compatibility of NASA's self-separation concept and TCAS, the mathematical definition of the volume determined by the SST is considered to be a conservative extension of the core TCAS II Resolution Advisory logic which checks against independent horizontal and vertical time and distance threshold.[2]

[2] Muñoz, Narkawicz, and Chamberlain, GNC, 2013.

Assumptions

- Two aircraft, the *ownship* and *intruder*,
- Accurate aircraft state information is available for both, i.e.,
  - Horizontal positions $\mathbf{s}_o, \mathbf{s}_i$ and velocities $\mathbf{v}_o$, $\mathbf{v}_i$
  - Altitudes s_{oz}, s_{iz} and vertical speeds v_{oz}, v_{iz}
  - Relative position $\mathbf{s} = \mathbf{s}_o - \mathbf{s}_i$ and velocity $\mathbf{v} = \mathbf{v}_o - \mathbf{v}_i$
  - Relative altitude $s_z = s_{oz} - s_{iz}$ and vertical speed $v_z = v_{oz} - v_{iz}$
- Prediction at a particular time instant of a future well-clear violation is based on a straight-line trajectory from that time instant, i.e., constant velocity is assumed.



A Family of Well-Clear Boundary Models

Definition of the Well Clear Volume

$$WCV_{t_{\mathrm{var}}}(\mathbf{s}, s_z, \mathbf{v}, v_z) \equiv \mathtt{Horizontal\_WCV}_{t_{\mathrm{var}}}(\mathbf{s}, \mathbf{v}) \text{ and } \mathtt{Vertical\_WCV}(s_z, v_z), \tag{1}$$

Anywhere inside the volume determined by this function, the aircraft are **not well clear**.

$$\begin{aligned}
\mathtt{Horizontal_WCV}_{t_{\mathrm{var}}}(\mathbf{s}, \mathbf{v}) &\equiv \|\mathbf{s}\| \leq \mathtt{DTHR} \text{ or} \\
&\quad (d_{\mathrm{cpa}}(\mathbf{s}, \mathbf{v}) \leq \mathtt{DTHR} \text{ and } 0 \leq t_{\mathrm{var}}(\mathbf{s}, \mathbf{v}) \leq \mathtt{TTHR}), \\
\mathtt{Vertical_WCV}(s_z, v_z) &\equiv |s_z| \leq \mathtt{ZTHR} \text{ or } 0 \leq t_{\mathrm{coa}}(s_z, v_z) \leq \mathtt{TCOA}.
\end{aligned}$$

$$\begin{aligned}
d_{\mathrm{cpa}}(\mathbf{s}, \mathbf{v}) &\equiv r(t_{\mathrm{cpa}}(\mathbf{s}, \mathbf{v})) = \|\mathbf{s} + t_{\mathrm{cpa}}(\mathbf{s}, \mathbf{v})\mathbf{v}\|, \\
\|s\| &\equiv \sqrt{\mathbf{s}^2} = \sqrt{\mathbf{s} \cdot \mathbf{s}} \\
|s_z| &\equiv s_{oz} - s_{iz}
\end{aligned}$$

A Family of Well-Clear Boundary Models

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\end{aligned} \tag{1}
$$

Anywhere inside the volume determined by this function, the aircraft are **not well clear**.

The function $t_{\text{var}}(\mathbf{s}, \mathbf{v})$ is the only change between the models

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|s_z| &\equiv s_{oz} - s_{iz}
\end{aligned}
$$

Parameter: Time Variables and Thresholds

Four choices for $t_{\text{var}}(\mathbf{s}, \mathbf{v})$:

$$\tau(\mathbf{s}, \mathbf{v}) \equiv \begin{cases} -\frac{\mathbf{s}^2}{\mathbf{s}\cdot\mathbf{v}} & \text{if } \mathbf{s}\cdot\mathbf{v} < 0, \\ -1 & \text{otherwise}, \end{cases} \tag{2}$$

$$t_{\text{cpa}}(\mathbf{s}, \mathbf{v}) \equiv \begin{cases} -\frac{\mathbf{s}\cdot\mathbf{v}}{\mathbf{v}^2} & \text{if } \mathbf{v} \neq \mathbf{0}, \\ 0 & \text{otherwise}, \end{cases} \tag{3}$$

$$\tau_{\text{mod}}(\mathbf{s}, \mathbf{v}) \equiv \begin{cases} \frac{\mathtt{DTHR}^2 - \mathbf{s}^2}{\mathbf{s}\cdot\mathbf{v}} & \text{if } \mathbf{s}\cdot\mathbf{v} < 0, \\ -1 & \text{otherwise}, \end{cases} \tag{4}$$

$$t_{\text{ep}}(\mathbf{s}, \mathbf{v}) \equiv \begin{cases} \Theta(\mathbf{s}, \mathbf{v}, \mathtt{DTHR}, -1) & \text{if } \mathbf{s}\cdot\mathbf{v} < 0 \text{ and } \Delta(\mathbf{s}, \mathbf{v}, \mathtt{DTHR}) \geq 0, \\ -1 & \text{otherwise}, \end{cases} \tag{5}$$

where

$$\Theta(\mathbf{s}, \mathbf{v}, D, \epsilon) \equiv \frac{-\mathbf{s}\cdot\mathbf{v} + \epsilon\sqrt{\Delta(\mathbf{s}, \mathbf{v}, D)}}{\mathbf{v}^2},$$

$$\Delta(\mathbf{s}, \mathbf{v}, D) \equiv D^2\mathbf{v}^2 - (\mathbf{s}\cdot\mathbf{v}^{\perp})^2.$$

All four models use the same vertical time variable to compare to `TCOA`:

$$t_{\text{coa}}(s_z, v_z) \equiv \begin{cases} -\frac{s_z}{v_z} & \text{if } s_z v_z < 0, \\ -1 & \text{otherwise}. \end{cases} \tag{6}$$

Parameter: Time Variables and Thresholds

Four choices for $t_{\text{var}}(\mathbf{s}, \mathbf{v})$:

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The four well clear volumes are in order of increasing containment

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Parameter: Time Variables and Thresholds, continued

$$\texttt{Horizontal\_WCV}_{t_{\mathsf{var}}}(\mathbf{s}, \mathbf{v}) \equiv \|\mathbf{s}\| \leq \texttt{DTHR} \text{ or }$$
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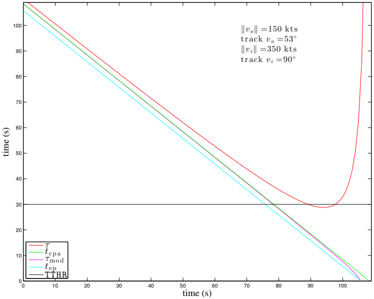


Figure : The 4 well clear volumes are in order of increasing containment

Conceptualizing the Well-Clear Boundary

- Sweep the ownship trajectory around 360° while holding v_{oz} constant,
- a boundary in three dimensions is determined by calling $WCV_{t_{\text{var}}}$ along each trajectory,
- project the resulting surface into the horizontal plane containing $\mathbf{s}_o$.

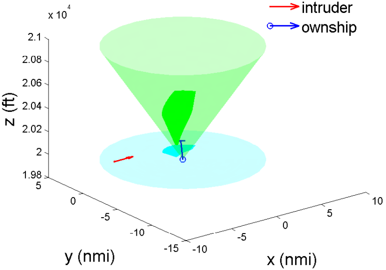
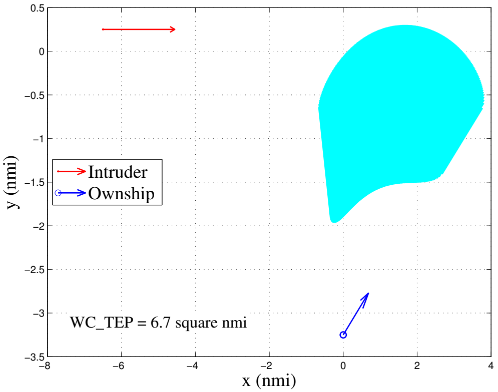


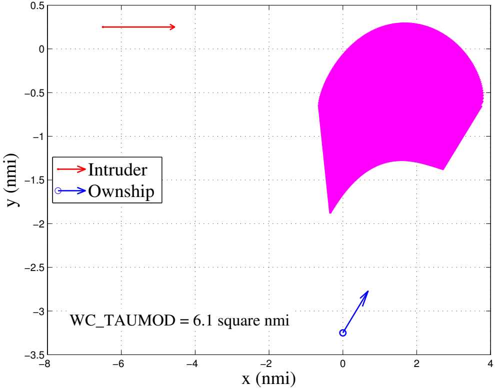
Figure : Illustration of a 3-dimensional encounter projected into 2 dimensions

WC_TEP



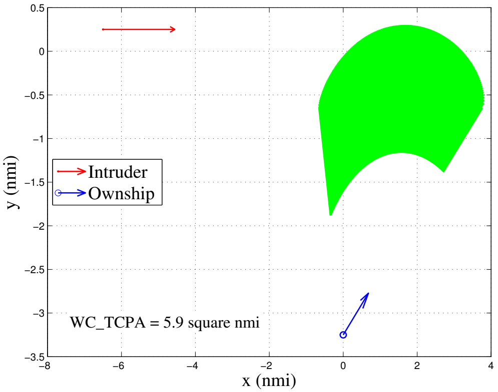
$$WCV_{t_{\text{ep}}}(\mathbf{s}, s_z, \mathbf{v}, v_z) \equiv \texttt{Horizontal\_WCV}_{t_{\text{ep}}}(\mathbf{s}, \mathbf{v}) \text{ and } \texttt{Vertical\_WCV}(s_z, v_z)$$

WC_TAUMOD



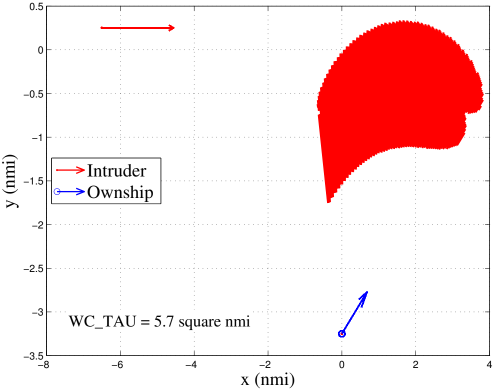
$$WCV_{\tau_{\text{mod}}}(\mathbf{s}, s_z, \mathbf{v}, v_z) \equiv \texttt{Horizontal\_WCV}_{\tau_{\text{mod}}}(\mathbf{s}, \mathbf{v}) \text{ and } \texttt{Vertical\_WCV}(s_z, v_z)$$

WC_TCPA



$$WCV_{t_{\text{cpa}}}(\mathbf{s}, s_z, \mathbf{v}, v_z) \equiv \texttt{Horizontal\_WCV}_{t_{\text{cpa}}}(\mathbf{s}, \mathbf{v}) \text{ and } \texttt{Vertical\_WCV}(s_z, v_z)$$

WC_TAU



$$WCV_\tau(\mathbf{s}, s_z, \mathbf{v}, v_z) \equiv \texttt{Horizontal\_WCV}_\tau(\mathbf{s}, \mathbf{v}) \text{ and } \texttt{Vertical\_WCV}(s_z, v_z)$$

Properties of Interest: Symmetry

Definition (Symmetry)

A well-clear boundary model specified by $WCV_{t_{var}}$, for a given time variable t_{var}, is symmetric *if and only if*

$$WCV_{t_{var}}(\mathbf{s}, s_z, \mathbf{v}, v_z) = WCV_{t_{var}}(-\mathbf{s}, -s_z, -\mathbf{v}, -v_z).$$

The ownship and intruder agree on whether they are well clear.

Theorem (Symmetry)

The well-clear boundary models WC_TAU, WC_TAUMOD, WC_TCPA, and WC_TEP are symmetric for any choice of threshold values `DTHR`, `TTHR`, `ZTHR`, and `TCOA`.

Properties of Interest: Inclusion

Theorem (Inclusion)

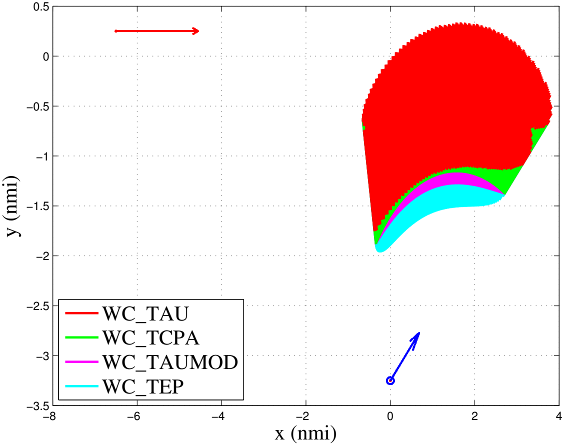
For all $\mathbf{s}, s_z, \mathbf{v}, v_z$ *and choice of threshold values* $\mathtt{DTHR}$, $\mathtt{TTHR}$, $\mathtt{ZTHR}$, *and* $\mathtt{TCOA}$, *the following implications hold*

(i) $WCV_{\tau}(\mathbf{s}, s_z, \mathbf{v}, v_z) \implies WCV_{t_{cpa}}(\mathbf{s}, s_z, \mathbf{v}, v_z)$,

(ii) $WCV_{t_{cpa}}(\mathbf{s}, s_z, \mathbf{v}, v_z) \implies WCV_{\tau_{mod}}(\mathbf{s}, s_z, \mathbf{v}, v_z)$, *and*

(iii) $WCV_{\tau_{mod}}(\mathbf{s}, s_z, \mathbf{v}, v_z) \implies WCV_{t_{ep}}(\mathbf{s}, s_z, \mathbf{v}, v_z)$.

Properties of Interest: Inclusion, continued



$$WCV_{t_{\mathrm{var}}}(\mathbf{s}, s_z, \mathbf{v}, v_z) \equiv \texttt{Horizontal\_WCV}_{t_{\mathrm{var}}}(\mathbf{s}, \mathbf{v}) \text{ and } \texttt{Vertical\_WCV}(s_z, v_z)$$

Properties of Interest: Local Convexity

A well-clear boundary model specified by $WCV_{t_{\text{var}}}$, for a given time variable t_{var}, is *locally convex* if and only if there are no times $0 \le t_1 \le t_2 \le t_3 \le T$ such that

1. the aircraft are not well clear at time t_1, i.e., $WCV_{t_{\text{var}}}(\mathbf{s} + t_1\mathbf{v}, s_z + t_1 v_z, \mathbf{v}, v_z)$,
2. the aircraft are well clear at time t_2, i.e., $\neg WCV_{t_{\text{var}}}(\mathbf{s} + t_2\mathbf{v}, s_z + t_2 v_z, \mathbf{v}, v_z)$, and
3. the aircraft not well clear at time t_3, i.e., $WCV_{t_{\text{var}}}(\mathbf{s} + t_3\mathbf{v}, s_z + t_3 v_z, \mathbf{v}, v_z)$.

Local Convexity: Along a linear trajectory, the aicraft does not lose well clear, gain it back, and lose it again.

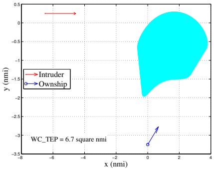


Figure : WC_TEP

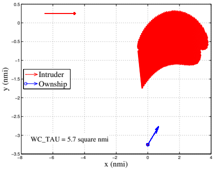


Figure : WC_TAU

Properties of Interest: Local Convexity, continued

Theorem

For any choice of threshold values, the well-clear boundary models WC_TCPA, WC_TAUMOD, and WC_TEP are locally convex.

Theorem

For some choices of threshold values, the well-clear boundary model WC_TAU is not locally convex.

Conclusion

- A formal definition of *well clear* is motivated by the need for UAS to operate safely in the presence of other aircraft in the airspace
- A family of well-clear boundary models is introduced which are extensions of the TCAS II RA logic
- Characterizing concepts for these models are:
  - Symmetry
  - Inclusion
  - Local convexity
- $\mathrm{WC\_TAU}$ has instances of non-local convexity and is the least conservative model
- $\mathrm{WC\_TEP}$ is the most conservative model

References

M. Consiglio, J. Chamberlain, C. Muñoz, and K. Hoffler, "Concept of integration for UAS operations in the NAS," in *Proceedings of 28th International Congress of the Aeronautical Sciences, ICAS 2012*, (Brisbane, Australia), 2012.

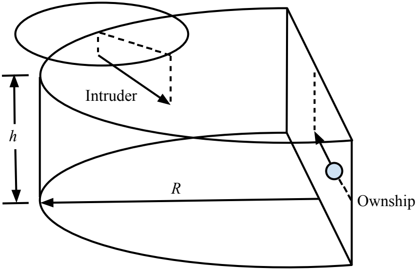
C. Muñoz, A. Narkawicz, and J. Chamberlain, "A TCAS-II resolution advisory detection algorithm," in *Proceedings of the AIAA Guidance Navigation, and Control Conference and Exhibit 2013, AIAA-2013-4622*, (Boston, Massachusetts), August 2013.

J. Upchurch, C. Muñoz, A. Narkawicz, J. Chamberlain, and M. Consiglio, "Analysis of well-clear boundary models for the integration of UAS in the NAS," NASA Technical Memorandum (submitted), 2014.

The End

Questions?

Encounter Space for Randomly-Generated Trajectories



- Ownship position, and horizontal direction fixed,
- Ownship and intruder horizontal velocity randomly chosen 849 velocities,
- Intruder horizontal position chosen from $\mathcal{U}[\pi, 2\pi]$,
- Intruder vertical position chosen from $\mathcal{N}(s_{oz}, h/6)$,
- Intruder horizontal velocity direction chosen from $\mathcal{U}[0, 2\pi]$,
- Intruder vertical velocity chosen from $\mathcal{N}(0, v_{iz,\max})$.

Example Encounters of Interest

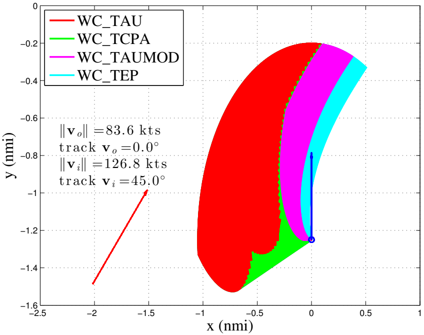


Figure : Large difference in t_{in}

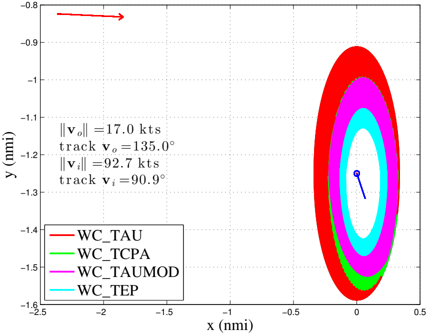


Figure : Disagreement in $WCV_{t_{\text{var}}}$